Lorentz violation induced vacuum birefringence and its astrophysical consequences*

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In the electromagnetism of loop quantum gravity, two helicities of a photon have different phase velocities and group velocities, termed as "vacuum birefringence". Two novel phenomenons, "peak doubling" and "de-polarization", are expected to appear for a linearly polarized light from astrophysical sources. We show that the criteria to observe these two phenomenons are the same. Further, from recently observed γ-ray polarization from Cygnus X-1, we obtain an upper limit $\sim 8.7 \times 10^{-12}$ for Lorentz-violating parameter χ, which is the most firm constraint from well-known systems. We also suggest to analyze possible existence of "peak doubling" through Fermi LAT GRBs.

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Symmetries are important ingredients in modern physics, and among which Lorentz symmetry is preeminently fundamental and profound. However, in searches of quantum gravity (QG), Lorentz violation (LV) emerges in many theoretical frameworks that try to conciliate apparent conflictions between standard model and general relativity [1–4]. As it offers a valuably observational window on QG, LV has stimulated lots of experimental works. Hitherto, LV parameters have been severely constrained from astrophysical observations and terrestrial experiments on various species [5–11]. However, no firm Lorentz-violating phenomenon has been confirmed yet.

Amongst phenomenological works, vacuum birefringence (VB), which shows great sensitivity to LV physics, is extensively studied [2, 3, 12–20]. VB is an analogy with birefringence within anisotropic medium, where left-handed and right-handed modes of light travel with different phase velocities and group velocities. It can arise from many parity-violating theories, including Chern-Simons terms [12, 21], loop quantum gravity [3, 13], Lorentz-violating effective field theories [2, 14, 15, 18, 19]. Lorentz-violating effects can modify phase velocities and group velocities of two oppositely circularly polarized modes, and they individually get a modification with an opposite sign. The modification is believed to be suppressed by some powers of the Planck length $l_{\rm Pl} \equiv \sqrt{G\hbar/c^3} \simeq 1.6 \times 10^{-35}$ m.[1]

As a consequence of VB, an originally linearly polarized light, which composes of left-handed and right-handed modes, from astrophysical and cosmological distance, will manifest "peak doubling" or "de-polarization" features when entering the observer [3, 13, 15–17, 20]. From this scenario, the Lorentz-violating parameter is constrained to a great precision from observations of polarized lights from the Crab Nebula [17] and γ-ray bursts (GRBs) [15, 16, 20]. In addition to probe LV physics, VB can also serve to distinguish parity-violating theories from those of even parity, like foamy spacetime [1] and doubly special relativity [4, 23].

In this report, we utilize the Lorentz-violating electromagnetism in loop quantum gravity [3]. By adopting an Ansatz accounting for differences in both phase velocities and group velocities, we obtain propagation behaviors and Stocks parameters of a linearly polarized light from cosmological distance. We show that the criteria to observe "peak doubling" and "de-polarization" are the same. By utilizing our derived formula to recently observed polarization of γ-rays from Cygnus X-1, we obtain an upper limit $\sim 8.7\times10^{-12}$ for Lorentz-violating parameter χ, which turns out to be the most firm constraint from well-known systems, though a little looser than that from the distance-estimated GRB 041219A [20]. Further, we re-propose the idea to analyze possible existence of peak doubling in light curves of most energetic Fermi LAT GRBs. In the paper, the convention $\hbar = c = 1$ is used.

In the picture of semi-classical spacetime with "polymer-like" structure that emerges in the loop quantum gravity, Gambini and Pullin derived the modified Maxwell equations [3], $\partial_t \vec{E} = \nabla \times \vec{B} + 2\chi l_{\rm Pl} \nabla^2 \vec{B}$ and $\partial_t \vec{B} = -\nabla \times \vec{E} - 2\chi l_{\rm Pl} \nabla^2 \vec{E}$, which break Lorentz boost symmetry as well as parity, while preserving rotation symmetry. From modified Maxwell equations, it is straightforward to get the modified dispersion relation for photons, $\Omega_\pm = |\vec{k}| \mp 2\chi l_{\rm Pl} |\vec{k}|^2$, where $\Omega_\pm$ are frequencies for left-handed and right-handed modes. A similar dispersion relation, $\Omega_\pm = |\vec{k}| \mp \xi l_{\rm Pl} |\vec{k}|^2$, can be attained from an effective field theory with Lorentz-violating dimension-5 operators for the photon sector [14].

Now from the modified dispersion relation, the phase velocity $v^{\rm p}$ and group velocity $v^{\rm g}$ of photons become

$$v^{\rm p}_\pm \equiv \frac{\Omega_\pm}{|\vec{k}|} = 1 \mp 2\chi l_{\rm Pl} |\vec{k}|, \ v^{\rm g}_\pm \equiv \frac{\partial \Omega_\pm}{\partial |\vec{k}|} = 1 \mp 4\chi l_{\rm Pl} |\vec{k}|, \quad (1)$$

respectively, and noticeably they are both helicity dependent, namely vacuum birefringent.

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[1] However, there are also arguments that a new fundamental scale might appear rather than the conventional Planck scale, see e.g., Ref. [22] and references therein.

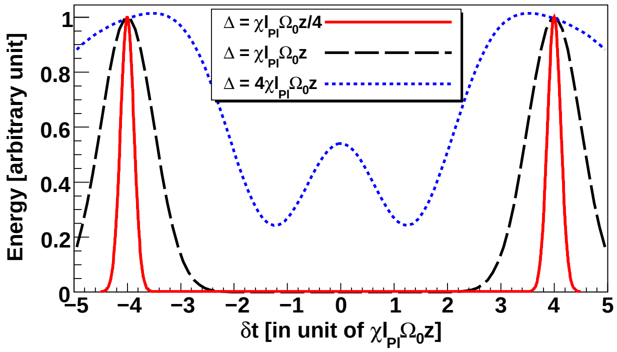

FIG. 1: The radiation arrives at the earth versus the "arrival time" δt.

We consider a fully linearly polarized light from astrophysical sources, whose electrical field $\vec{E}$ is a superposition of two monochromatic waves with opposite circular polarizations, i.e., $\vec{E} = \vec{E}_+(k_+) + \vec{E}_-(k_-)$. The radiation can be produced from various mechanisms, e.g., through synchrotron radiation in a region penetrated with well ordered magnetic fields. This can be achieved in the vicinity of a neutron star, of an active galactic nucleus (AGN), and of a GRB. For a photon traveling along z-axis with its central frequency Ω_0, the wavenumbers for two modes are $k_\pm = \Omega_0(1 \pm 2\chi l_{\rm Pl}\Omega_0)$. Assuming a Gaussian wave packet with a width Δ in space, we have [13]

$$\vec{E}_\pm \propto \mathrm{Re}\left\{\exp\left[i(\Omega_0 t - k_\pm z)\right]\exp\left[-\frac{(z - v_\pm^{\rm g} t)^2}{\Delta^2}\right]\hat{\rm e}_\pm\right\}, \tag{2}$$

where $\hat{\rm e}_\pm \equiv \hat{\rm e}_1 \pm i\hat{\rm e}_2$.

Conventionally, two modes arrive at the earth at the same time $t_0 = z$, hence we are able to detect a superposition, i.e., a linearly polarized light. However, with LV effects, VB is induced, and their times of arrival can be different. To focus on the detection epoch, let us notate $z = t + \delta t$. Now bigger δt means earlier arrival.

Ref. [3] noticed that these two modes will be separated by a distance $\sim 8\chi l_{\rm Pl}\Omega_0 z$, hence they arrive at the earth in sequence. The energy, $\propto |\vec{E}|^2$, arrives at the earth versus the "arrival time" δt is illustrated in Fig. 1 for three different widths in space, $\Delta = \chi l_{\rm Pl}\Omega_0 z/4$, $\chi l_{\rm Pl}\Omega_0 z$, and $4\chi l_{\rm Pl}\Omega_0 z$. We adopt $\chi l_{\rm Pl}\Omega_0^2 z = 1$ in the calculation. Typically, we have $\chi l_{\rm Pl}\Omega_0^2 z \simeq 4.8\frac{\chi}{10^{-14}}\frac{l_{\rm Pl}}{10^{-28}\ {\rm eV}^{-1}}(\frac{\Omega_0}{100\ {\rm keV}})^2\frac{z}{10^{10}\ {\rm l.y.}}$. For comparisons, we also calculate $\chi l_{\rm Pl}\Omega_0^2 z = 10$ and $\chi l_{\rm Pl}\Omega_0^2 z = 100$ cases.[2] It is shown that the profile is merely modified. From the figure, we can see that "peak doubling" appears when the width Δ is small. With increasing Δ, two peaks tend to merge into one single signal. Hence to detect such a phenomenon, the width of packet should be smaller than the doubling separation, i.e., $\chi l_{\rm Pl}\Omega_0 z > \Delta$.

We can also easily get canonical Stocks parameters for the light.[2] After averaging over time, the degree of linear polarization $\varpi_L = \exp\left[-32\chi^2 l_{\rm Pl}^2\Omega_0^2 z^2/\Delta^2\right]$, which is exponentially suppressed. Hence the original polarization will be smeared drastically if $\chi l_{\rm Pl}\Omega_0 z > \Delta$, termed as "de-polarization". This criterion turns out to be the same as that for "peak doubling".

Most previous "de-polarization" analysis bases on a reasoning that, the observation of polarization indicates that the rotated angle caused by VB between photons with low energy Ω_{0l}, and those with high energy Ω_{0h}, is smaller than π, i.e., $|2\chi l_{\rm Pl}(\Omega_{0h}^2 - \Omega_{0l}^2)z| \lesssim \pi$. This is absolutely plausible when the spectrum is flat, however, when it deviates from a flat one, essential cautions should be kept in mind, especially when it is steep. The observed polarization may be due to dominant low energy photons, because the contribution from high energy ones can be largely suppressed and contributes insubstantially. In most realistic cases in astrophysics, the spectrum turns to be decreasing as a function of energy, $N(\Omega_0) \propto \Omega_0^{-\Gamma}{\rm e}^{-\Omega_0/E_0}$, where E_0 denotes a possible cutoff, and Γ varies according to different radiation mechanisms and electron distributions. Typically, $\Gamma = 1 \sim 4$ for X/γ-rays. Therefore, the contribution from high energy photons is indeed minor, compared to the large population of low energy ones. Cautions should be kept in mind when the cutoff is obvious and/or the measured polarization is small, say $\lesssim 5\%$. On the other hand, a more convincing result can be drawn after taking the population of photons into account and convoluting it with the Ansatz presented here.

TABLE I: Some previous observational constraints from astrophysics. Numerical factors between our notation from those of previous literatures on $|\chi|_{\rm upper}$ are accounted.

Source	z	Energy	$\varpi_L(\%)$	$\|\chi\|_{\rm upper}$
3C 256	$\mathcal{Z}\simeq1.82$	3000-4000 Å	16.4±2.2	5 10^{-5} [13]
GRB 020813	$\mathcal{Z}\simeq1.3$	3500-8800 Å	1.8-2.4	1 10^{-7} [16]
GRB 021004	$\mathcal{Z}\simeq2.3$	3500-8600 Å	$\lesssim 2$	5 10^{-8} [16]
GRB 021206	$\mathcal{Z}\sim0.1$	0.15-2.0 MeV	80±20	1 10^{-15} [15]
Crab pulsar	$\sim$2 kpc	0.1-1 MeV	46±10	2 10^{-10} [17]
GRB 041219A	$\mathcal{Z}\sim0.3$	100-350 keV	63^{+31}_{-30}, 96^{+39}_{-40}	1 10^{-14} [20]

In Table I, we list six constraints determined previously, where three utilize optical/ultraviolet lights, while the other use γ-rays. Because the rotated angle depends quadratically on the energy and only linearly on the distance, high energy observations have a big advantage. It can be seen clearly from the table that the highest energy observation, GRB 021206, could place the most stringent constraint [15]. However, the observation is refuted later [24, 25]. Hence the most stringent constraint comes from GRB 041219A, whose "pseudo-redshift" was estimated to be $\mathcal{Z} \sim 0.3$ by Stecker and the Lorentz-violating parameter is determined to be $\chi \lesssim 10^{-14}$ [20].

[2] See arXiv:1104.4438v1 [astro-ph.HE] for details.

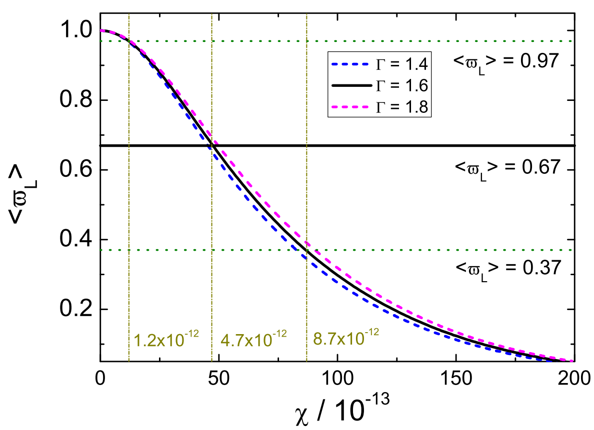

FIG. 2: The spectrally averaged degree of linear polarization of Cygnus X-1 γ-ray emissions, versus the Lorentz-violating parameter χ.

It is significant, though the estimated distance of GRB 041219A needs further verification. In addition, polarized observation from the Crab Nebula constrained firmly on χ to be $\lesssim 2 \times 10^{-10}$ [17].

Recent INTEGRAL/IBIS observation of Cygnus X-1 black hole binary system found evidently that the γ-ray emission is largely polarized with $\langle \varpi_L \rangle = 67 \pm 30\%$ in the energy band 400 keV–2 MeV [26]. The spectrally averaged degree of linear polarization is defined as $\langle \varpi_L \rangle \equiv \int N(\Omega_0)\varpi_L(\Omega_0)\mathrm{d}\Omega_0 / \int N(\Omega_0)\mathrm{d}\Omega_0$, where $N(\Omega_0) \propto E^{-\Gamma}$ is given by the INTEGRAL observation with a photon index $\Gamma = 1.6 \pm 0.2$ [26]. We can roughly estimate the LV parameter χ to be around $\sim 10^{-11}$ with a reasonable width in space $\Delta \sim 1\ \mathrm{MeV}^{-1}$, and observational parameter $z = 2.1$ kpc, $\Omega_0 \sim 1$ MeV. To be more precise, we also perform a detailed analysis taking the spectral effects into account. The result is shown in Fig. 2, where, as a conservative treatment, 100% polarization at source is assumed. We can see that the photon index Γ has minor effects on the conclusion, so we would use its central value $\Gamma = 1.6$ in the following. Through the observed value $\langle \varpi_L \rangle = 67 \pm 30\%$, we can infer an upper limit on χ. The most conservative situation gives $|\chi|_{\mathrm{upper}} = 8.7 \times 10^{-12}$.

Our upper limit $|\chi|_{\mathrm{upper}} = 8.7 \times 10^{-12}$ is better than the upper limit from the Crab pulsar, though somehow looser than that inferred from GRB 041219A. Worthy to mention that, the difference between Cygnus X-1 and GRB 041219A mainly comes from distances. However, in the GRB 041219A case, the distance is estimated through "pseudo-redshift", not from direct observations. In contrast, Cygnus X-1 binary is a much well-known system in astronomy, hence our upper limit on χ, though seems looser, actually bases on more firm observations.

Our "de-polarization" criterion is somehow different, and only when $\Delta \simeq 1/\Omega_0$, it turns into $\chi l_{\mathrm{Pl}} \Omega_0^2 z > 1$, then seems similar to that used previously. However, Δ is not promised to equal to $1/\Omega_0$, instead, it should be around $1/\delta\Omega_0$, where $\delta\Omega_0$ is the uncertainty of the energy of photons, according to Heisenberg's uncertainty principle. Considering the radiation mechanism of astrophysical sources, $\delta\Omega_0$ reflects the "fuzziness" of the process. The "fuzziness" is determined by the environmental conditions when generating the light, including the irregularities of magnetic fields, the distribution of electrons, and quantum mechanical effects. Astrophysicists are putting great efforts to get these quantities from various observations and inferences. In order to estimate our criterion, we rewrite it into $\frac{\chi}{10^{-14}} \frac{l_{\mathrm{Pl}}}{10^{-28}\ \mathrm{eV}^{-1}} \frac{\Omega_0}{100\ \mathrm{keV}} \frac{\delta\Omega_0}{20\ \mathrm{keV}} \frac{z}{10^{10}\ \mathrm{l.y.}} > 1$. We can see that with an organized environment which provides $\delta\Omega_0 \sim 20$ keV, our criterion is met if we observe a polarized light of $\gtrsim 100$ keV from cosmological distance when $\chi \sim 10^{-14}$. These conditions are already practical nowadays, therefore, we are expected to observe or further constrain Lorentz-violating birefringence from astronomical observations. With upcoming γ-ray polarimetric instruments, POET (POlarimeters for Energetic Transients) [27] for an example, we can study VB better to validate/falsify relevant QG theories.

When VB was considered in the context of loop quantum gravity [3], BATSE was the best performing satellite in detecting GRBs. Hence, concerning technical parameters of BATSE, Gambini and Pullin considered a fictitious GRB at a cosmological distance, $z \sim 10^{10}$ light years, and $\Omega_0 \sim 200$ keV, which produces a doubling $\sim 10^{-5}$ s if χ is of order $\mathcal{O}(1)$ [3]. However, this is out of detectability then. Therefore, main interests of studies of VB were grasped into possible observations of "de-polarization", for it seems more sensitive to Lorentz-violating parameters.

Nowadays, however, there are several reasons to reconsider studies of the possible existence of peak doubling. First, as discussed above, "de-polarization" studies can sometimes have problematic explanations when regarding the non-uniform population of photons induced by the spectrum. Second, at the time of BATSE, the observed peak width of GRBs appears to be of order ~ 0.1 s, with features like a rising edge ~ 1 ms [3]. In contrast, after about ten years, current Fermi LAT instrument has timing accuracy $< 10\ \mu$s, and maximum energy detectability up to 300 GeV [28]. Third, peak doubling and de-polarization correspond to differences in group velocity and phase velocity, respectively. There are still debates on theoretical predictions of these two velocities. Hence, as a different approach to falsify/verify LV from de-polarization, peak doubling has its own irreplaceable significance in LV searches. Even if LV is confirmed from de-polarization observations, it is still largely valuable to detect peak doubling as a consistent check or an additional study. In addition, in the high energy band, peak doubling has extra observational merits, compared with de-polarization. Technically, timing measurement is somehow easier than polarization measurement in the

high energy band. To pindown polarization properties, we should have enough events for statistics, in contrast, timing of detection of high energy photons relays less on statistics. From another point of view, the doubling signal will be buried in the sea of photons if the photon flux is too large, and fortunately, at high energies, we can analyze it more neatly.

Concerning technical progresses made in recent years, we rewrite the timing separation of peak doubling into $8\chi l_{\rm Pl}\Omega_0 z \sim 10^2\chi \frac{l_{\rm Pl}}{10^{-28}\ {\rm eV}^{-1}} \frac{\Omega_0}{300\ {\rm GeV}} \frac{z}{10^{10}\ {\rm l.y.}}$ s. With timing accuracy < 10 μs of Fermi LAT, we can have sensitivity down to $\chi \sim \mathcal{O}(10^{-7})$. This is of the same order of that from de-polarization constraints from GRB 020813 and GRB 021004 [16], though still several orders away from X/γ-ray GRB observations (see Table I for comparisons). Worthy to mention that, our criterion is automatically satisfied with Fermi parameters. With eight orders of improvement over the past ten years, we would be positive to have extra improvements of magnitudes to detect possible existence of peak doubling in the future through next generation of satellites, or through multi-TeV photon observations from ground-based cosmic-ray observatories.

In summary, Lorentz-violating and parity-violating quantum gravitational theories predict vacuum birefringence, where two circularly polarized modes of a linearly polarized light have different phase velocities and group velocities. Hence, an originally linearly polarized light produced in astrophysical processes can manifest new phenomenons when arriving at the observer after traveling through a cosmological distance. "Peak doubling" and "de-polarization" are expected to be observed with a non-vanishing Lorentz-violating parameter of a suitable magnitude. Inversely, non-observations of these two phenomenons can be used to constrain the Lorentz-violating parameter.

In this report, we study an Ansatz which concerns both differences in phase velocities and group velocities of two oppositely circularly polarized modes in the electromagnetism of loop quantum gravity. "Peak doubling" and "de-polarization" phenomenons can be easily read from our analysis. We also present criteria to observe "peak doubling" and "de-polarization" from astrophysical studies, and these two criteria turn out to be the same. Comparisons of theoretical criteria and observational practicality are also presented with some emphasis on possible Fermi LAT observations of "peak doubling", which is rarely discussed in literatures. We also analyze the recently observed polarization of γ-ray emissions from Cygnus X-1 black hole, and obtain an upper limit $\chi \lesssim 8.7 \times 10^{-12}$, which turns out to be the best constraint from well-known systems. We would expect the quantum-gravitationally induced vacuum birefringence issue to meet a more distinguishable situation from upcoming observations of newly planned astronomical instruments, POET for an example, with abilities to measure polarizations of high energy photons, or new instruments, Fermi LAT for an example, with precision to detect rapid variability in the high energy γ-ray light curves.

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